Harmonic Motion with Springs

1) A 13kg box is attached to a spring & then compressed 0.3m to the left. (A) What is the spring constant if a force of 60N is required to compress the spring 0.3m? (B) What force is the spring applying to the box when it reaches its equilibrium position? (C) What & where is the maximum velocity of the box? (D) What & where is the maximum acceleration of the box? (E) What is the velocity & acceleration of the box when the spring is stretched 0.2m?

(A) Applied force $-60N$
$F_{spring} = +60N$

$F_s = -kx$
$60 = -k(-.3)$
$\boxed{k = 200 N/m}$

(B) $\boxed{0N}$

(C) max. vel. = at equilibrium

$\frac{1}{2}kA^2 = \frac{1}{2}mv^2$

$\frac{1}{2}(200)(.3)^2 = \frac{1}{2}(13)v^2$
$9 = 6.5v^2$
$\boxed{v = \pm 1.18 m/s}$

(D) Max acc. is when Force is the greatest
$\Rightarrow$ amplitude $(\pm .3m)$

$F = ma$
$\pm 60 = 13a$

$\boxed{a = \pm 4.62 m/s^2}$

(E) velocity

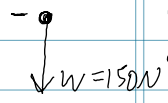
$9 = \frac{1}{2}kA^2 + \frac{1}{2}mv^2$
$9 = \frac{1}{2}(200)(.2)^2 + \frac{1}{2}(13)v^2$
$9 = 4 + 6.5v^2$
$v = \pm .88 m/s$

Acc.
$F = ma$ & $F = -kx$

$-(200)(\pm .2) = 13a$

$\boxed{a = -3.08 m/s^2}$

2) A spring is attached to the ceiling and a 15 kg mass is also attached. Assuming friction is negligable, (A) what is the spring constant of the spring if the mass is released at the spring's equilibrium position & it falls 1.2m before it stops & then accelerates up undergoing harmonic motion? (B) What is the maximum force & acceleration of the mass & at what location? (C) What is the maximum velocity of the mass & at what location? (D) How fast is it moving when it has fallen 0.2m?

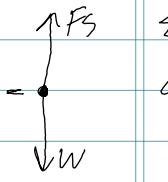


$v=0$
$a=max.$
$\Sigma F=ma$

(A) $mgh = \frac{1}{2}kx^2$
$15(10)(1.2) = \frac{1}{2}(k)(1.2)^2$
$180 = .72k$
$\boxed{k = 250\,N/m}$

(B) Location ⇒ at amplitude
ie ⇒ height $\boxed{0m \text{ or } 1.2m}$

Top ⇒ $\Sigma F = W = ma$
$-150 = 15a$ $\boxed{a = -10\,m/s^2}$
$\boxed{150N}$

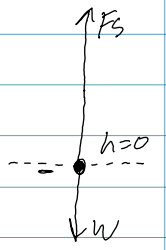


$\Sigma F = 0$
$a = 0$
$v = max.$

$\Sigma F_{eq} = 0 = F_s + W$
$F_s = 150$

Bottom ⇒ $F_s = -kx = -250(-1.2)$
$F_s = 300N$

$F_s = -kx$
$150 = -k(-.6)$
$\boxed{k = 250N/m}$



$\Sigma F = F_s + W = ma$
$300 - 150 = 15(a)$
$\boxed{150N} = 15a$ $\boxed{a = +10\,m/s^2}$

$v=0$
$a=max.$
$\Sigma F=max.$

(C) Location is at equil.
$\boxed{x = .6m}$

(D)
$180 = \frac{1}{2}(250)(.2)^2 + 15(10)(1) + \frac{1}{2}(15)v^2$
$180 = 5 + 150 + 7.5v^2$
$25 = 7.5v^2$
$v^2 = 3.33$

$\boxed{v = -1.83\,m/s}$

$mgh = \frac{1}{2}kx^2 + mgh + \frac{1}{2}mv^2$
$15(10)(1.2) = \frac{1}{2}(250)(.6)^2 + 15(10)(.6) + \frac{1}{2}(15)v^2$
$180 = 45 + 90 + 7.5v^2$
$90 = 45 + 7.5v^2$
$45 = 7.5v^2$
$v^2 = 6$
$\boxed{v = \pm 2.45\,m/s}$