

Harmonic Motion with Springs

- 1) A 13kg box is attached to a spring & then compressed 0.3m to the left. (A) What is the spring constant if a force of 60N is required to compress the spring 0.3m? (B) What force is the spring applying to the box when it reaches its equilibrium position? (C) What & where is the maximum velocity of the box? (D) What & where is the maximum acceleration of the box? (E) What is the velocity & acceleration of the box when the spring is stretched 0.2m?

(A) Applied force = 60N
 $F_{\text{spring}} = +60\text{N}$

$$F_s = -kx$$
$$60 = -k(-.3)$$
$$k = 200\text{N/m}$$

(B) 0N

(C) max. vel. = at equilibrium

$$\frac{1}{2}kA^2 = \frac{1}{2}mv^2$$

$$\frac{1}{2}(200)(.3)^2 = \frac{1}{2}(13)v^2$$
$$q = 6.5v^2$$
$$v = \pm 1.18\text{m/s}$$

(D) Max acc. is when
Force is the greatest
⇒ amplitude ($\pm .3\text{m}$)

$$F = ma$$
$$\pm 60 = 13a$$

$$a = \pm 4.62\text{m/s}^2$$

(E) velocity

$$q = \frac{1}{2}kA^2 + \frac{1}{2}mv^2$$
$$q = \frac{1}{2}(200)(.2)^2 + \frac{1}{2}(13)v^2$$
$$q = 4 + 6.5v^2$$
$$v = \pm .88\text{m/s}$$

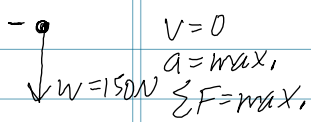
Acc.

$$F = ma \quad \& \quad F = -kx$$

$$-(200)(\pm .2) = 13a$$

$$a = -3.08\text{m/s}^2$$

2) A spring is attached to the ceiling and a 15 kg mass is also attached. Assuming friction is negligible, (A) what is the spring constant of the spring if the mass is released at the spring's equilibrium position & it falls 1.2 m before it stops & then accelerates up undergoing harmonic motion? (B) What is the maximum force & acceleration of the mass & at what location? (C) What is the maximum velocity of the mass & at what location? (D) How fast is it moving when it has fallen 0.2 m?



$$(A) mgh = \frac{1}{2}kx^2$$

$$15(10)(1.2) = \frac{1}{2}(k)(1.2)^2$$

$$180 = .72k$$

$$k = 250 \text{ N/m}$$

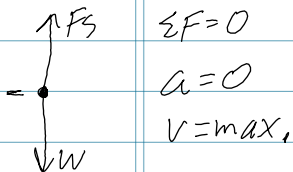
OR

(B) Location \rightarrow at amplitude
ie \Rightarrow height 0 m or 1.2 m

Top $\Rightarrow \Sigma F = W = ma$

$$\uparrow -150 = 15a \quad \boxed{a = -10 \text{ m/s}^2}$$

$$\boxed{150 \text{ N}}$$



$$\Sigma F_{eq} = 0 = F_s + W$$

$$F_s = 150$$

Bottom $\Rightarrow F_s = -kx = -250(-1.2)$
 $F_s = 300 \text{ N}$

$$F_s = -kx$$

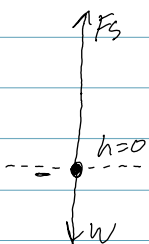
$$150 = -k(-.6)$$

$$k = 250 \text{ N/m}$$

$$\Sigma F = F_s + W = ma$$

$$300 - 150 = 15(a)$$

$$\boxed{150 \text{ N}} = 15a \quad \boxed{a = +10 \text{ m/s}^2}$$



(C) Location is at equilo

$$\boxed{x = .6 \text{ m}}$$

(D)

$$180 = \frac{1}{2}(250)(.2)^2 + 15(10)(1) + \frac{1}{2}(15)v^2$$

$$180 = 5 + 150 + 7.5v^2$$

$$25 = 7.5v^2$$

$$v^2 = 3.33\bar{3}$$

$$\boxed{v = -1.83 \text{ m/s}}$$

$$mgh = \frac{1}{2}kx^2 + \{mgh\} + \frac{1}{2}mv^2$$

$$15(10)(1.2) = \frac{1}{2}(250)(.6)^2 + 15(10)(.6) + \frac{1}{2}(15)v^2$$

$$180 = 45 + 90 + 7.5v^2$$

$$90 = 45 + 7.5v^2$$

$$45 = 7.5v^2$$

$$v^2 = 6$$

$$\boxed{v = \pm 2.45 \text{ m/s}}$$