

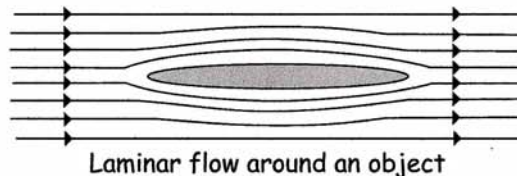
AP Physics – Moving Fluids

There are two types of flow that fluids can undergo; Laminar flow and *turbulent flow*.

Laminar flow is also known as streamline flow.

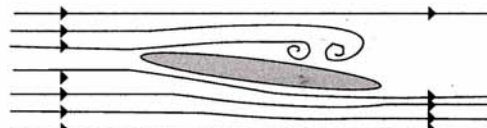
Laminar flow is the motion of a fluid in which

every particle in the fluid follows the same path as that followed by previous particles. Basically it means that the particles in the fluid are traveling in smooth lines, one right after the other. We can represent laminar flow by drawing *streamlines*, which are example paths that the fluid particles are traveling along.



Laminar flow around an object

Turbulent flow occurs at high flow rates and when the fluid is moving past irregular shapes. In turbulent flow, the motion of the fluid becomes chaotic, and it forms eddies and whirlpools. Turbulent flow absorbs energy and increases the frictional drag throughout the fluid.



Turbulent flow around an object

Turbulent flow is very complicated and the actual motion of the fluid cannot be precisely calculated. The mathematics needed to even approximate the flow is pretty hairy (and not all that accurate, models are used which come fairly close to describing the behavior, but the models can be off).

We will deal mainly with laminar flow and ignore turbulent flow.

In dealing with flow through pipes and tubes, we will assume that the fluid is incompressible (pretty close for liquids, they pretty much are incompressible, but not exactly perfect for gas flow, gases being subject, as you remember from chemistry, being compressible). We will also assume that they encounter no internal friction as they flow through the pipe.

Rate of Flow: The *rate of flow* is defined as the volume of fluid that passes a certain cross section in a given time. In mathematical terms, this rate of flow is:

$$\boxed{R = vA}$$

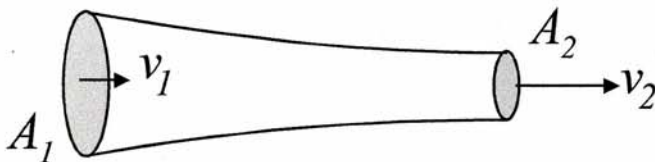
NOT given on the AP exam but part of an equation that is given

Where R is the flow rate and v is the velocity of the fluid. A is the cross-sectional area of the pipe.

Common units for rate of flow are cubic feet per second, m³/s, gallons per second, liters per second, etc. Almost any volume unit and almost time unit can be used to express flow rate.

Unit 1 Notes 3 Moving Fluids

The **flow rate** must be a **constant throughout** the length of the pipe, as we are ignoring friction and assuming that the fluid cannot be compressed. What **goes in** has got to be what **come out the other end**. Imagine water entering a hose at one end, traveling through the hose, and then coming out of the other end of the hose. The water that enters the hose in a given time has to equal the water that leaves the hose in the same time. So **R**, the flow rate, remains **constant** no matter what happens inside the hose.



** The flow rate will be constant even if the radius of the pipe changes **

Upstream, the cross-sectional area, A_1 , is larger than the cross-sectional area downstream, A_2 . The flow rate at both of these points must be the same. The flow rate is:

$$R = vA$$

so

$$R = v_1 A_1 = v_2 A_2$$

v_1 is the velocity of the fluid upstream, v_2 is the flow rate downstream, A_1 is the upstream cross-sectional area, and A_2 is the downstream cross-sectional area.

$$\text{So } v_1 A_1 = v_2 A_2$$

This equation is give on the AP Exam

Practice Problem:

- 1) Water flows through a rubber hose 2.0 cm in diameter at a velocity of 4.0 m/s. If the hose is coupled into a hose that has a diameter of 3.5 cm, what is the new speed of the fluid?

$$v_1 A_1 = v_2 A_2$$

$$v_2 = \frac{v_1 A_1}{A_2} = \frac{4 \cancel{\pi} (.01)^2}{\cancel{\pi} (.0175)^2} = 1.31 \text{ m/s}$$

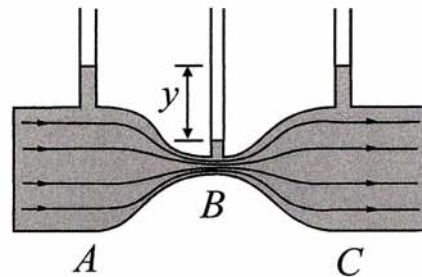
The Nozzle Effect: When a fluid flows through a narrow opening, a nozzle, its velocity must increase. We can see this in the following problem.

- 2) Water flows through a rubber hose 3.0 cm in diameter at a velocity of 5.0 m/s. If the hose is coupled into a nozzle that has a diameter of 0.50 cm, what is the new speed of the fluid?

$$v_2 = \frac{v_1 A_1}{A_2} = \frac{5 \cancel{\pi} (.015)^2}{\cancel{\pi} (.0025)^2} = 180 \text{ m/s}$$

Unit 1 Notes 3 Moving Fluids

Bernoulli's Principle: A fluid's velocity increases you have seen, when it flows through a constriction - the diameter of the pipe decreases. To accelerate the fluid as it goes into the constriction, the pushing force in the large diameter area must be greater than the pushing force in the constriction.



This is shown in the drawing above. We have a horizontal pipe that narrows and then resumes its original diameter. Attached at *A*, *B*, and *C* are small tubes filled with fluid. The height of the liquid in these tubes indicates their relative pressures. At *A* and *C* the pressure is greater than it is at *B*. If *y* is the difference in height between the liquid columns, then the pressure difference is given by:

$$P_A - P_B = \rho gy$$

This change in pressure that takes place in a constriction is called the Venturi effect. The Venturi effect says that pressure changes are accompanied by changes in velocity.

In the 1700's Daniel Bernoulli (1700 – 1782), a Swiss scientist, experimented with water flowing through pipes. He found that the pressure exerted by a liquid on its walls decreased as its velocity increased. He found it to be true for both liquids and gases. Today we call this the *Bernoulli principle*. In simple form, Bernoulli's principle says this:

When the speed of a liquid increases, its internal pressure decreases.

This is a consequence of the conservation of energy. Bernoulli developed an equation which relates pressure and velocity in a fluid system which is called Bernoulli's equation.

$$P + \rho gy + \frac{1}{2} \rho v^2 = \text{const.} \quad \text{This equation is the last equation given on the exam!!!}$$

Where *const.* is some constant, *P* is the pressure, ρ is the fluid density, *y* is the height of the fluid, and *v* is the fluid velocity.

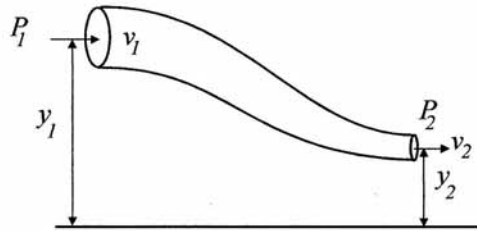
The ρgy term is the potential energy per unit volume of the flowing fluid.

The $\frac{1}{2} \rho v^2$ is the kinetic energy per unit volume of the flowing fluid.

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We can analyze the flow of a fluid through a system using this equation.

If we look at two locations in the system, we know that the sum of the pressure, kinetic energy, and potential energy have to equal a constant, i.e., they have to be the same, so we can write:



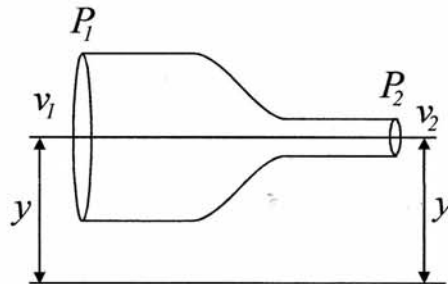
$$P_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2$$

(NOT GIVEN ON THE EXAM but think conservation of energy)

Water flows through a pipe that has a constriction in it as shown. Using the equation we developed above

$$P_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2$$

Analyzing the terms in Bernoulli's equation, we see that the potential energy remains the same and cancels out (no change in height, y). Therefore:



$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

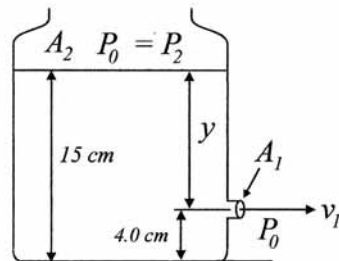
$$P_2 = P_1 + \frac{1}{2} \rho v_1^2 - \frac{1}{2} \rho v_2^2 = P_1 + \frac{1}{2} \rho (v_1^2 - v_2^2)$$

The initial velocity is smaller than the final velocity, so P_2 has to be less than P_1 .

Let's use Bernoulli's equation to examine a static system.

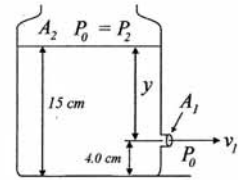
Practice Problem:

- 3) A container of water, diameter 12 cm, has a small opening near the bottom that can be unplugged so that the water can run out. If the top of the tank is open to the atmosphere, what is the exit speed of the water leaving through the hole. The water level is 15 cm above the bottom of the container. The center of the 3.0 diameter hole is 4.0 cm from the bottom.



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Let's figure what is going on with the flow out the hole at the bottom. We have the area of the hole A_1 and the area of the container A_2 .



The water spurts out of the hole with a speed of v_1 . The flow of water in the container, which makes the surface level drop is very slow by comparison. So slow that we can say that it is zero. So $v_2 = 0$. **Important idea here !!!! **

The pressure on the top of the surface is the atmospheric pressure. The surface acting on the water at the bottom is also the atmospheric pressure (actually it is a tiny bit bigger because it is slightly lower, but the difference is insignificant). So we can let the two pressures equal each other. So $P_1 = P_2$.

Therefore, Bernoulli's equation,

$$P_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2$$

Becomes:

$$\rho g y_1 + \frac{1}{2} \rho v_1^2 = \rho g y_2$$

This simplifies to:

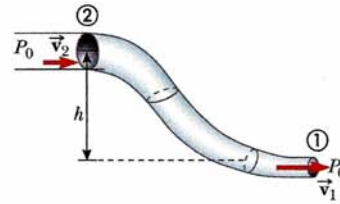
$$g y_1 + \frac{1}{2} v_1^2 = g y_2$$

We can plug in the data:

$$10(.04) + \frac{1}{2} v_1^2 = 10(.15) \quad \frac{1}{2} v_1^2 = 1.1 \quad \boxed{v_1 = 1.48 \text{ m/s}}$$

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- 4) A large pipe with a cross sectional area of 1.0 m^2 descends 5.0 m and narrows to 0.50 m^2 , where it terminates in a valve at point 1. If the pressure at point 2 is atmospheric pressure, and the valve is opened wide and water allowed to flow freely, find the speed of the water leaving the pipe.



Both sides are again open so $P_1 = P_2 = P_0$ and can be cancelled out of the equation

$$P_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2$$

If you look closely, there are too many unknowns!!
We don't know v_1 or v_2 so we need another equation

$$A_1 v_1 = A_2 v_2 \quad v_2 = \frac{A_1 v_1}{A_2}$$

Lets put those two equations together

$$\frac{1}{2} \rho v_1^2 = \rho g y_2 + \frac{1}{2} \rho \left(\frac{A_1 v_1}{A_2} \right)^2 \quad \frac{1}{2} v_1^2 = g y_2 + \frac{1}{2} \left(\frac{A_1 v_1}{A_2} \right)^2$$

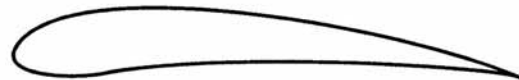
And now plug in our values

$$\frac{1}{2} v_1^2 = (10)(5) + \frac{1}{2} \left(\frac{.5(v_1)}{1} \right)^2 \quad v_1^2 = 100 + .25 v_1^2 \quad v_1 = 11.55 \text{ m/s}$$

Applications of Bernoulli's Principle: The Venturi effect is well illustrated in the classic hose nozzle. Water, if allowed to pour out of the end of a one-inch hose, is not traveling at much of a speed. Attach a nozzle to the thing. The nozzle makes the water squeeze through a very small opening. As it does this, its velocity increases greatly, and the stream shoots out like crazy. Its pressure decreases as it goes through the nozzle. This pressure drop does not immediately make sense, because you know that the stream of water blasting out of the nozzle can put a hurt on stuff you go squirting at. Do not confuse the pressure that a liquid has within it with the pressure it can exert when something interferes with its flow. The pressure *within* a shooting stream of water is low, but the pressure it can exert on something in its path can be quite large. The force it exerts on things in its path is a result of the kinetic energy it has and its momentum.

Bernoulli and Flight: Bernoulli's principle is often used to explain why birds and airplanes can fly. One of the reasons that their flight is possible is because of the shape of their wings and the way that air flows over and under these wings. Birds evolved the proper shape for a wing at least 135 million years ago. Human beings didn't catch on until fairly recently.

In the 1880s a British scientist named George Cayley developed the cambered wing. It shared a cross-sectional shape with the wing of birds. It looks like this:



Cambered Wing