

## Newtonian Mechanics

$$1) \quad \text{a) } v = v_0 + at \qquad \text{b) } x = x_0 + v_0t + \frac{1}{2}at^2 \qquad \text{c) } v^2 = v_0^2 + 2a(x - x_0)$$

**Basic equations of motion** – used in most kinematics problems. Equation c useful for problems where time is not known. May need to use combination of these equations to solve the problem. Note that for free-fall problems  $a = -g$ . For projectile problems, separate into x and y dimensions and solve separately. Remember that velocity and acceleration are vectors.

$$2) \quad a = \frac{\sum F}{m} = \frac{F_{Net}}{m}$$

**Newton's second law of motion.** Remember that force and acceleration are vectors. If multiple forces are present, solve independently for each force and combine for vector sum. Use free body diagrams to determine the net force – don't try to solve for acceleration until you have determined the net force via vector methods.

$$3) \quad |F_f| \leq \mu |F_N|$$

**Force of friction** depends on the coefficient of friction ( $\mu$ ) and the normal force (N). Normal force is always perpendicular to the surface in question – e.g. need to use trigonometry to find N for inclined plane. Note that ( $\mu$ ) is usually different for kinetic (moving) friction and for static (stationary) friction – it takes more force to start an object moving than to keep it moving.

$$4) \quad a_c = \frac{v^2}{r}$$

**Centripetal acceleration.** Direction is always toward the center of the circle of radius r. ( $v$ ) is the tangential speed at any point. Note that for rotational motion, the magnitude of the velocity is constant, but acceleration  $\neq 0$  since the direction of the velocity is changing.

$$5) \quad \tau = rF \sin \theta$$

**Torque equation.** Here r is the distance from force to object and F is magnitude of the force.  $r \sin \theta$  is known as the “moment arm” distance. In many problems, the force is perpendicular to r and  $\sin \theta = 1$  so the moment arm = r and  $\tau = rF$ .

$$6) \quad \mathbf{p} = m\mathbf{v}$$

**Definition of linear momentum.** Remember that momentum is a vector quantity, with the direction determined by the direction of the velocity. Remember law of conservation of momentum which is very useful in solving collision problems. Linear momentum is always conserved during collisions unless there are external forces acting.

$$7) \quad \mathbf{J} = \mathbf{F}\Delta t = \Delta\mathbf{p}$$

**Definition of Impulse.** Note that impulse is a vector quantity with direction determined by the direction of the force or direction of momentum. Impulse is useful in solving problems where the force acts for a short time, but is not constant – here the change in momentum equals the average force times the time. Typical examples would include bat hitting a baseball, ball bouncing off wall, etc.

$$8) K = \frac{1}{2}mv^2$$

**Definition of kinetic energy.** Note that energy (in general) is a scalar quantity. Kinetic energy is scalar even though it is a function of a vector quantity (velocity). Remember the law of conservation of energy – kinetic and potential energy (next equation) calculations can be used to solve many mechanics problems in a simple and elegant way.

$$9) \Delta U_g = mgh$$

**Gravitational potential energy.** Potential energy is defined as the ability to do work. As an object falls, the gravitational potential energy is converted to kinetic energy – you can use this equivalence to determine the speed of a falling object as a function of distance fallen.

$$10) W = \mathbf{F} \cdot \Delta \mathbf{r} = F\Delta r \cos \theta$$

**Definition of work.** Work is a measure of energy and is a scalar even though it is a function of two vectors. Note that total work done over any closed path in a conservative force field (like a gravitational or electric field) is zero – you may see a question pertaining to this. That means no work is done on an object undergoing uniform circular motion.

$$11) P_{avg} = \frac{W}{\Delta t}$$

**Definition of power.** You can use this equation to determine energy produced (or consumed) if you know the average power and the time in question.

$$12) P = \mathbf{F} \cdot \mathbf{v} = Fv \cos \theta$$

**Power.** This equation is useful for determining the power taken to maintain a given speed – typical problem would be to determine the power it takes for a car to maintain a certain speed while climbing a hill of a certain pitch.

$$13) \mathbf{F}_s = -k\mathbf{x}$$

**Force exerted by a spring.** It's important to remember the force is not constant, but is always changing. It's at a maximum at the amplitude and a minimum at the equilibrium point.

$$14) U_s = \frac{1}{2}kx^2$$

**Potential energy of a stretched (or compressed) spring.** As a spring system oscillates the potential energy is converted into kinetic energy – this can be used to compute the velocity at any point in the oscillation.

$$15) T_s = 2\pi\sqrt{\frac{m}{k}}$$

**Period of oscillation of a spring system.** Note that the period doesn't depend on the amount of displacement.

$$16) T_p = 2\pi\sqrt{\frac{l}{g}}$$

**Period of a pendulum.** Note that the period is independent of the mass of the pendulum and the arc of the swing (true for relatively small arcs).

$$17) T = \frac{1}{f}$$

**Period vs frequency for any type of periodic motion.** This expression applies to wave motion, simple harmonic motion (pendulum, spring) etc.

$$18) F_G = -\frac{Gm_1m_2}{r^2}$$

**Newton's law of gravitation.** Useful in celestial mechanics calculations – what is the weight of something on the moon?, etc. Note that this expression provides another way to determine g (acceleration due to gravity). Remember that the distance r is the distance between centers of the objects and that you can generally assume that all the mass of the object is concentrated at its center.

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$$18) U_G = -\frac{Gm_1m_2}{r}$$

**General expression for gravitational potential energy.** Used in celestial mechanics calculations (large objects (planets) and large distances). In calculations near the surface of the earth equation 9 ( $\Delta U_g = mgh$ ) is more useful.

## Electricity and Magnetism

$$19) F = k \frac{q_1q_2}{r^2}$$

**Coulomb's Law for the electrostatic force.** Note that this is an inverse square force ( $F \propto \frac{1}{r^2}$ ), just like the gravitational force. The electrostatic force is conservative – the total work done around any closed path in the force field is zero. In many of our problems we used the expression  $F = k \frac{q_1q_2}{r^2}$  where  $k = 9 \times 10^9 \text{ N m}^2/\text{C}^2$ .

$$20) I_{avg} = \frac{\Delta Q}{\Delta t}$$

**Definition of the electric current.** Electric current is defined as charge/unit time or Coulombs/second. Current is usually measured in Amps (1 Amp = 1 Coulomb/second). If you are given a problem stating the flow of electrons in a conductor, you can determine the current with this expression.

$$21) R = \frac{\rho \ell}{A}$$

**Electrical resistance of a substance.** Here  $\rho$  represents the resistivity of the substance (ohms/meter), A is the cross-sectional area of the substance and  $\ell$  is the length. In AP test problems, you will be either given the value of  $\rho$  or asked to compute it, given other factors.

$$22) V = IR$$

**Ohm's law.** The basic equation of electronics, relating voltage (electric potential) to current and resistance. Ohm's law allows you to solve for V, I, or R, given 2 of the 3 variables. Simple circuits can usually be solved by using Ohm's law and the equations for series and parallel resistors (equations 33 and 34). More complicated circuits (e.g. multiple voltage sources) usually must be solved using **Kirchoff's Rules** in addition to Ohm's law.

Note that Kirchoff's Rules are not shown on the AP equation sheet, so you need to remember them – fortunately, they're pretty simple:

**Kirchoff's Junction Rule:** sum of all currents entering or leaving any junction is zero (or total current in = total current out).

**Kirchoff's Loop Rule:** sum of all voltage drops around any continuous circuit loop = zero.

23)  $P = IV$

**Power in an electric circuit.** Remember to use the correct values for  $I$  and  $V$  when using this equation – e.g. when determining the power dissipated by a single resistor in a larger circuit,  $I$  is the current through the resistor (not necessarily the total current in the circuit) and  $V$  is the voltage difference across the resistor (not the total voltage of the circuit). In many cases, a more useful expression for the power dissipated by a resistor is  $P = I^2 R$ , where  $I$  is the current through the resistor and  $R$  is the resistance.

24)  $R_s = \sum_i R_i$

**Total resistance of a group of resistors connected in series.** Note that this is a similar expression to that for a group of parallel capacitors.

25)  $\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

**Total resistance of a group of resistors connected in parallel.** Note that this is a similar expression to that for a group of series capacitors. Two additional equations (not shown on the AP sheet) are simpler and quicker for special cases (these equations can be derived from the general equation).

a) If only two resistors are connected in parallel then  $R_p = \frac{R_1 R_2}{R_1 + R_2}$

b) If all parallel resistors are equal then  $R_p = \frac{R}{n}$  where  $n$  = number of parallel resistors and  $R$  is the resistance of each resistor.

## Waves and Optics

26)  $v = f\lambda$

**Velocity of a wave.** This is a fundamental wave equation that relates the velocity of a wave to its frequency and wavelength. One implication of the equation is that if frequency is constant and the wave velocity changes, the wavelength must change – this has implications in optics.