AP Physics - Harmonic Motion

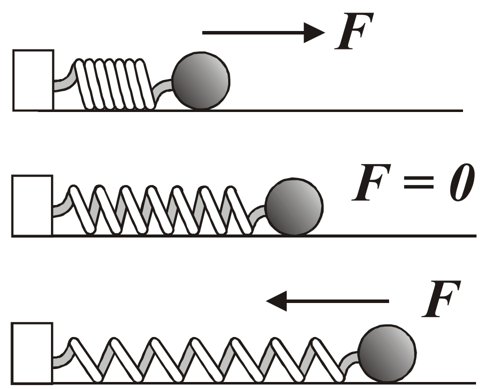
We have been dealing with straight line motion or motion that is circular. There are other types of motion that must be dealt with, specifically a motion that repeats itself over and over. Such a thing has its very own phancy physics name, ***periodic motion***.

> ***Periodic Motion → motion in which a body moves back and forth with a fixed path over a definite interval of time.***

A special case of periodic motion is known as ***harmonic motion***.

> ***Harmonic Motion → Periodic motion with no friction that is produced by a restoring force that is directly proportional to the displacement and oppositely directed.***

Don't you hate definitions like that? Let's see how it works!



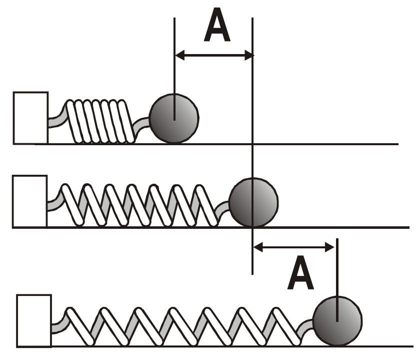
Let's look at a simple example of harmonic motion. A sphere is attached by a spring to a solid block. The sphere (and spring) is free to slide back and forth on a smooth, frictionless surface.

Initially the ________________ is _________________________. This is shown in the first drawing. The ball is released and the ____. In the second drawing, the spring has reached its ____________________________________ and is ______ longer exerting a __________________ on the ball. Ah, but the ball's inertia and the good old first law ensure that the ball keeps moving. As it does this, it stretches the spring. Of course this causes the spring to exert a _________________________________ ___________________which slows the ball. This is the ___________________________________. Anyway, eventually the spring force has had enough time to stop the ball. This is depicted in the third drawing. Once the ___________________________, the spring will __________________________ in the opposite direction and the process will repeat itself.

Here are some critical terms that have to do with such motion.

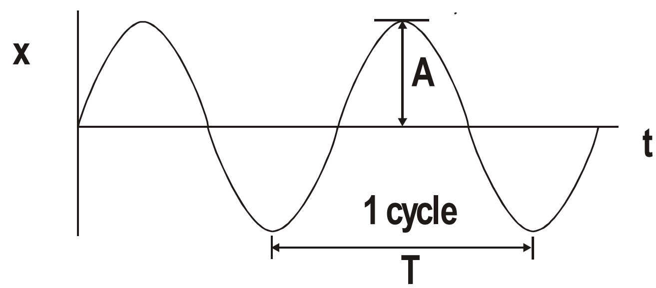
- ***Equilibrium position → this is the point in the motion where the object would be if it were not subject to any forces.***
- ***Amplitude (A) → The maximum displacement from the equilibrium position.***
- ***Cycle → One complete iteration of the motion sequence.***
- ***Period (T) → The time it takes to do one cycle.***
- ***Frequency (f) → The number of cycles per unit of time.***

The drawing to the right gives us a depiction of the amplitude of the system.



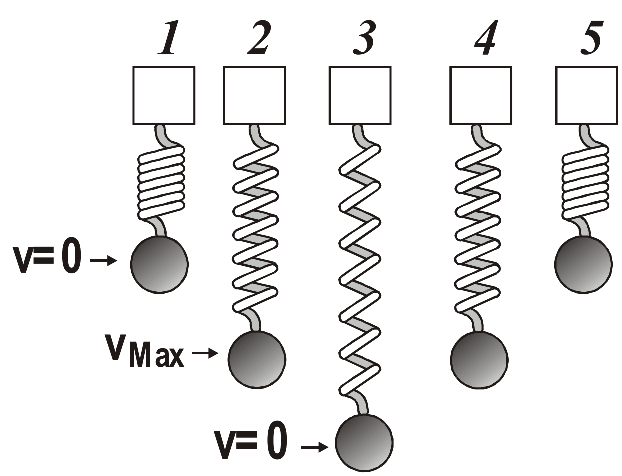
• Notice the ______________________ is always in the__________________________ direction of the __________________________________

Another way to look at this motion is to look at a graph of displacement vs time. The graph looks like this:



The ___________________________ is plotted along the ___ axis and _________________ is plotted along the___ axis. The ______________________________ is the ____ value. On the graph, __________________________ is the segment of the curve from one _______________________________ to the ______________ sequential _____________________________________. These are points along the displacement path where the object is doing the same thing.

No better way to understand the thing than to look at another example. Here we have a weight attached to a vertically mounted spring. The weight is given an initial small displacement (which will be the amplitude) and then released. It bobs up and down. Its __________________ will ________________ as it moves through the cycle. The velocity is __________________________________ of the motion __________ at the _______________________. The _____________________________________ occurs at the ____ position.



Acceleration and displacement: We can analyze the motion more fully by looking at the forces in the system, the accelerations that are taking place, and the displacement.

We begin with Hooke's law, which we studied when we dealt with energy. This is the force that is exerted by the spring.

Hooke's Law $F_s = -kx$

Recall that the ______________________________ simply means that the ______________________ exerted ______________________________ is always _____________________ to the ________________________.

The ____________________________ will occur at the __________________________________, which is the amplitude, so we can write an equation for the maximum force.

$$F_{Max} = -kA$$

Where $\boldsymbol{F_{Max}}$ is the maximum restoring force, $\boldsymbol{k}$ is the spring constant and $\boldsymbol{A}$ is the amplitude.

We can now apply the second law to find the maximum acceleration.

$$F_{Max} = ma_{Max} \qquad \text{or} \qquad a_{Max} = \frac{F_{Max}}{m}$$

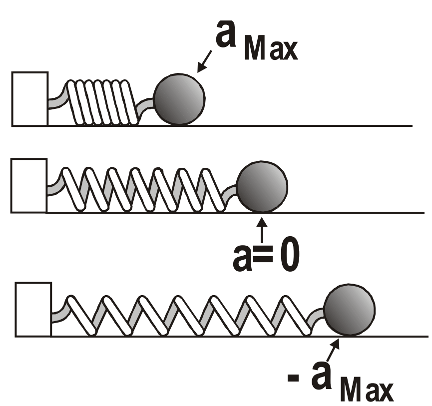
We can plug $F_{Max} = -kA$ into the acceleration equation we just developed:

$$a_{Max} = \frac{F_{Max}}{m} \quad = (-kA)\frac{1}{m} \quad = -\frac{kA}{m} \qquad \text{so} \quad a_{Max} = -\frac{kA}{m}$$

The ___________________ is __________________________ because the ____________________ is ______ _____________________. We can write an equation for the ____________________ as a ________________ of ______________________________.

<u> Not give on the formula sheet **</u>**

At ______________________________, the _________________ will be _______________, since, as you can see, the displacement x is zero. The acceleration is varying then between its maximum value, which takes place at the maximum displacement (the good old amplitude) and zero.



The acceleration ranges from $-\frac{k}{m}A$ to $+\frac{k}{m}A$. In between it is given by: $a = -\frac{k}{m}x$

In the drawing above, we can see the positions that have the maximum acceleration. We have chosen the coordinate system so that acceleration to the right is positive and acceleration to the left is negative.

Another important idea is that the ______________________________________ given the system – we could call this the _______________________________ – will ______________________ the _________________. So you pull the spring out and give it a displacement of 5.0 cm, that means that the amplitude has to be 5.0 cm.

Example Problem

- A spring has a constant of 125 N/m. A 350.0 g block is attached to it and is free to slide horizontally on a smooth surface. You give the block an initial displacement of 7.00 cm. What is (a) the maximum force and (b) the maximum acceleration acting on the block?

(a) To find the force we use Hooke's law:

(b) To find the acceleration we use the second law:

- A 13,000 N car starts at rest and rolls down a hill from a height of 10.0 m. It then moves across a level surface and collides with a light spring loaded guard rail. A) Neglecting friction, find the maximum distance the spring is compressed (k=1.0 X 106 N/m). B) Calculate the maximum acceleration of the car after it contacts with the spring?

(a) To find the spring compression we use the conservation of energy from the previous unit:

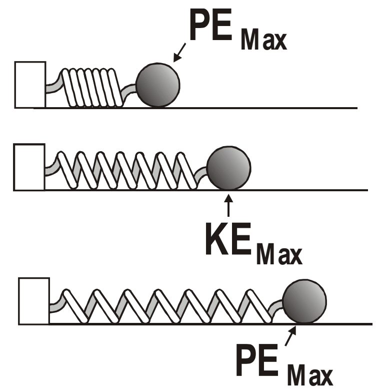
(b) To find the max acceleration we use the derived equation that includes acc. & springs:

Remember the Relationship Between Frequency and Period:

The frequency of the system is the number of cycles per unit time. The unit for frequency is the Hertz.

$$f = \frac{number\ of\ cycles}{time} = \frac{1\,cycle}{Period} \quad \text{so} \quad f = \frac{1}{T} \quad \text{or} \quad T = \frac{1}{f}$$

Energy and Amplitude: The ______________________________ of a system undergoing harmonic motion must, by law, __________________ ____________________. But of course the _________________ of energy __________________________________ from one type or types to another.



Clearly the energy of the system must be _____________________________ _______________when the ______________________ is at its __________ value. This is because the velocity of the system is zero. This will occur when the _______________________ is ____. Once the mass is moving away from maximum displacement, ________________ of the potential _________________ is __________________________ to ____________________ energy. The ____________________ energy _____________________ and the __________________________ energy ______________________ until the displacement is zero. At this point _______ the energy ________________________ and the object is _____________________ at _______ __________________. The kinetic energy then begins to decrease and the potential energy increases until the amplitude is reached again. And so on.

The potential energy for a spring is given by this equation

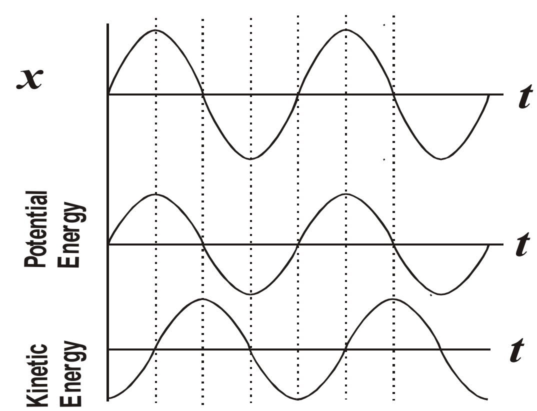
$$U_S = \frac{1}{2}kx^2$$ change the "x" to "A" and you get

This is the maximum amount of energy that the system can have – the system will get no other energy.

The Law of Conservation of Energy tells us that the total energy of the system must remain constant, so this equation gives us the energy that the system has at any point in the cycle.

Also not that the ______________________ is ___________________________ to the _________________ of the ______________________________.

We can plot kinetic energy *vs.* time, potential energy *vs.* time, and displacement *vs.* time and then compare them.



*** You should be prepared to mark up a drawing or a graph of a harmonic motion situation and locate the places where the kinetic energy is at a maximum or minimum, the potential energy's maximum and minimum values, where the velocity is zero or maximum, where the acceleration is minimum or maximum, and where the forces are minimum or maximum.

Calculating Energy: From the law of conservation of energy, we know that the energy must stay constant for the system This means that:

$$(K+U+U_S)_i=(K+U+U_S)_f$$

But since we are ignoring gravitational potential energy, the expression is simplified to:

$$(K+U_S)_i=(K+U_S)_f$$

We also know that the total energy of the system must be:

$$U_S=\frac{1}{2}kA^2$$

The energy of the system for any displacement must be:

$$\frac{1}{2}kA^2=\frac{1}{2}mv^2+\frac{1}{2}kx^2$$

$$kA^2=mv^2+kx^2$$

Example Problem

- A 0.50 kg object connected to a light spring with a spring constant of 20 N/m oscillates on a frictionless horizontal surface. A) Calculate the total energy of the system and the maximum speed of the object if the amplitude of the motion is 3.0 cm. B) What is the velocity of the object when the displacement is 2.0 cm? C) Compute the kinetic energy and potential energies of the system when the displacement is 2.0 cm.

(a) To find the total energy and the max. speed we can use the conservation of energy:

(b) To find the velocity of the object at a position of 2.0 cm we still use energy but remember to include elastic energy on the final side:

(c) Continue to use your energy equations:

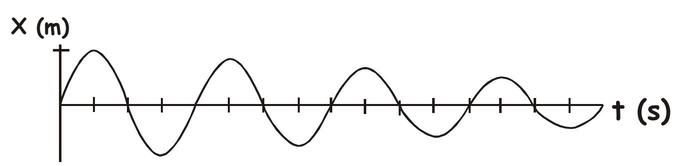
Period of spring system: The period for a spring system moving with a small displacement is given by this equation. (You will be provided with this equation on the AP Physics Test).

T is the period, ***m*** is the mass of the weight (we are ignoring the mass of the spring, as it is usually insignificant), and ***k*** is the spring constant.

- A 345 g block is part of a spring system, is oscillating. The spring constant is 125 N/m. What is the period of the system?

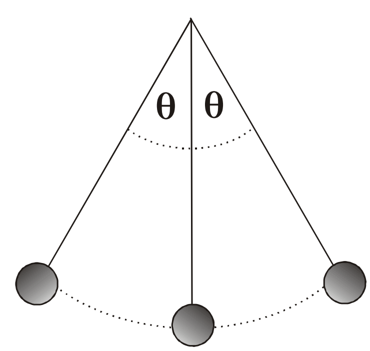
- A 1300 kg car is constructed on a frame supported by four springs. Each spring has a spring constant of 20,000 N/m. If 2 people riding in the car have a combined mass of 160 kg, find the frequency and period of vibration of the car when it is driven over a pothole in the road.

In the real world, we have to deal with friction and energy losses, so that the actual graph would look like this:



The____, although the __________________ and frequency ______________________________________. This decrease in amplitude deal is called _____________________________ or ***attenuation*** (takes your pick). We will, in solving our little problems, ignore this dampening effect.

Period of a Pendulum: Another simple harmonic system is a pendulum. The period of a pendulum is constant for small angle swings (for large angle swings, the period is not a constant).

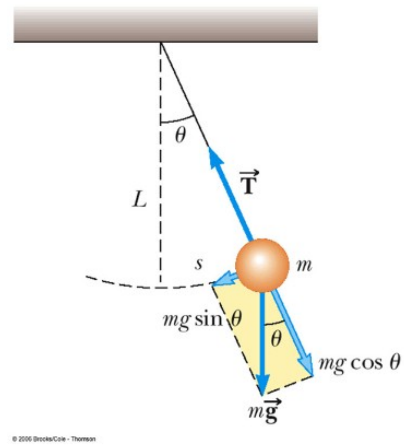


The period of a pendulum is given by this formula (which will also be available to you on the AP Physics Test).

T is the period, ***L*** is the length of the pendulum, and ***g*** is the acceleration of gravity.

**Note that the _______________ of a pendulum __________________ only on its _________________ **

Notice the ________________________ is the ______________________ of the _____________________ tangent to the path of motion. The net is therefore equal to:



Pendulums are commonly used to experimentally find a value for *g*, the acceleration of gravity in physics experiments. How would you go about doing that?

- A tall tower has a cable attached to the ceiling with a heavy weight suspended at its bottom near the floor. If the frequency of the pendulum is 0.0667 Hz, how long is the cable?

  We first convert the freq. to period and then use the period equation and solve it for the length.