Advanced Placement Physics Circular Motion, Gravity, & Satellites

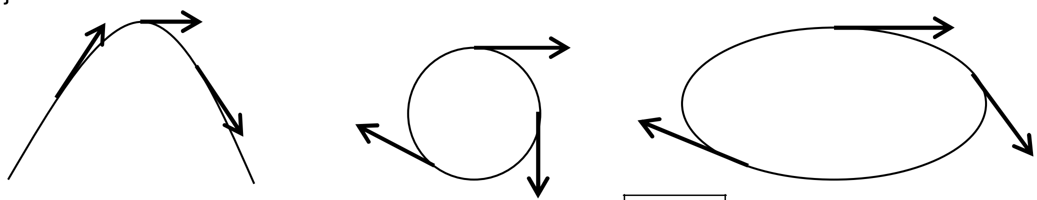
Frequency: How often a repeating event happens. Measured in revolutions per second. Time is in the denominator.

Period: The time for one revolution. $T = \frac{1}{f}$ Time is in the numerator. It is the inverse of frequency.

Speed: Traveling in circles requires speed since direction is changing.

Velocity: However, you can measure instantaneous velocity for a point on the curve. Instantaneous velocity in any type of curved motion is tangent to the curve. ***Tangential Velocity***.

Projectile Motion Circular Motion Satellite Motion



The equation for speed and tangential velocity is the same $v = \frac{2\pi r}{T}$

Acceleration: ***Centripetal Acceleration.*** Due to inertia objects would follow the tangential velocity. But, they don't. The direction is being changed toward the center of the circle, or to the foci. In other words they are being accelerated toward the center. $a_c = \frac{v^2}{r}$ Centripetal means center seeking.

Force: ***Centripetal Force.*** If an object is changing direction (accelerating) it must be doing so because a force is acting. Remember objects follow inertia (in this case the tangential velocity) unless acted upon by an external force. If the object is changing direction to the center of the circle or to the foci it must be forced that way. $F_c = ma_c$ $F_c = m\frac{v^2}{r}$

1. **As always, ask what the object is doing.** Changing direction, accelerating, toward the center, force centripetal.
2. **Set the direction of motion as positive.** Toward the center is positive, since this is the desired outcome.
3. **Identify the sum of force equation.** In circular motion F_c is the sum of force. F_c can be any of the previous forces. If gravity is causing circular motion then $F_c = F_g$. If friction is then $F_c = F_{fr}$. If a surface is then $F_c = F_N$.
4. **Substitute the relevant force equations and solve.** For F_c substitute $m\frac{v^2}{r}$

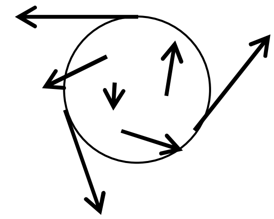
Gravity $F_g = G\frac{m_1 m_2}{r^2}$ ***and*** $F_g = mg$ ***combined are*** $mg = G\frac{m_1 m_2}{r^2}$ ***simplified is*** $g = G\frac{m}{r^2}$

r is not a radius, but is the distance between attracting objects measured from center to center. Is the problem asking for the height of a satellite above earth's surface? After you get ***r*** from the equation subtract earth's radius. Are you given height above the surface? Add the earth's radius to get ***r*** and then plug this in. **Think center to center**.

Inverse Square Law: If ***r*** doubles (x2), invert to get ½ and then square it to get ¼. Gravity is ¼ its original value so F_g is ¼ of what it was and ***g*** is ¼ of what it was. So multiply F_g by ¼ to get the new weight, or multiply ***g*** by ¼ to get the new acceleration of gravity. If ***r*** is cut to a (x 1/3), invert it to get 3 and square it to get 9. Multiply F_g or ***g*** by 9.

Apparent Weight: This is a consequence of your inertia. When an elevator, jet airplane, rocket, etc. accelerates upward the passenger wants to stay put due to inertia and is pulled down by gravity. The elevator pushes up and you feel heavier. Add the acceleration of the elevator to the acceleration of gravity $F_{g\ apparent} = mg + ma$. If the elevator is going down subtract $F_{g\ apparent} = mg - ma$. If the elevator is falling you will feel weightless $g = a\ so\ F_{g\ apparent} = 0$. This same phenomenon works in circular motion. Your inertia wants to send you flying at the tangential velocity. You feel pressed up against the side of the car on the outside of the turn. So you think there is a force directed outward. This false non-existent force is really your inertia trying to send you out of the circle. The side of the car keeps you in moving in a circle just as the floor of the elevator moves you up. The car is forced to the center of the turn. No force exists to the outside. However, it feels like gravity, just like your inertia in the accelerating elevator makes you feel heavier. You are feeling ***g's*** similar to what fighter pilots feel when turning hard. It is not your real weight, but rather what you appear to weight, ***apparent weight***.

Rotation



All parts of an object are rotating around the axis. All parts of the body have the same period of rotation. This means that the parts farther from the central axis of rotation are moving faster. So if we look at some of the tangential velocities diagramed at the right, we see that they are in all directions and vary in magnitude. So we need a new measurement of velocity. Collectively all the velocities are known as the ***angular velocity***, which is a measure of the radians turned by the object per second. Because the period is the same for the various parts of the rotating object, they move through the same angle in the same time. In rotation the parts of a rotating body on the outside move faster. They need to travel through the same number of degree or radians in the same amount of time as the inner parts of the body, but the circumference near the edge of a spinning object is longer than close to the center. So the outer edge must be moving faster to cover the longer distance in the same time interval. (This differs from the circular motion of the planets, which are not attached, and therefore not a single rotating body. The planets move in circular motion individually. Here the inner planets move faster. The planets closer to the sun must move faster in order to escape the gravity of the sun. They also travel a shorter distance and therefore have the shortest period of orbit).

> ***All the equations for an object in circular motion hold true if we are looking at a single point and only a specific point on a rotating object.***

Rotating objects have ***rotational inertia*** and an accompanying ***angular momentum***, meaning that a rotating object will continue to rotate unless acted upon by an ***unbalanced torque***, & a non-rotating object will not rotate unless acted upon by an ***unbalanced torque***.

Torque: The force that causes rotation. In rotation problems we look at the sum of torque (not the sum of force). But it is exactly the same methodology.

$$\boxed{\tau = rF\sin\theta}$$

Strongest when the force is ***perpendicular*** to the lever arm (since sin 90º equals one).

Balanced Torque: The sum of torque is zero. No rotation.

Unbalance Torque: Adding all the clockwise and counterclockwise torque does not sum to zero. So there is excess torque in either the clockwise or counterclockwise direction. This will cause the object to rotate.

1. **As always, ask what the object is doing.** Is it rotating or is it standing still?
2. **Set the direction of motion as positive.** It will either rotate clockwise or counterclockwise. If you pick the wrong direction your final answer will be negative, telling you that you did thing in reverse. But, the answer will be correct nonetheless. If it is not moving pick one direction to be positive, it really doesn't matter. But the other must be negative, so that the torque cancels.
3. **Identify the sum of torque equation.**

$$\sum\tau = \sum\tau_{clockwsise} - \sum\tau_{counterclockwsise} \quad \textbf{\textit{or}} \quad \sum\tau = \sum\tau_{counterclockwsise} - \sum\tau_{clockwsise}$$

4. **Substitute the relevant force equations and solve** (examples assume clockwise was positive direction)

Rotating you get some + / - $\sum\tau = \sum\left(rF\sin\theta\right)_{clockwsise} - \sum\left(rF\sin\theta\right)_{counterclockwsise}$

Not Rotating $0 = \sum\left(rF\sin\theta\right)_{clockwsise} - \sum\left(rF\sin\theta\right)_{counterclockwsise}$